Exam answers, Labour Economics, June 2019

1 Labor supply and taxation

1. The worker's problem is given by

$$\begin{aligned}\max_{c,l} U(c,l) &= c - \frac{\gamma}{\eta}(l_0 - l)^{\eta} \\ \text{s.t. } c &\geq w(l_0 - l) \\ 0 &\leq l \geq l_0\end{aligned}$$

Since the utility function is monotone in consumption we must have that the worker consumes all of her income, that is $c = w(l_0 - l)$. The Lagrangian is given by

$$L = c - \frac{\gamma}{\eta}(l_0 - l)^{\eta} + \mu(w(l_0 - l) - c)$$

$$\frac{\partial L}{\partial c} = 0 \Leftrightarrow \mu = 1$$

$$\frac{\partial L}{\partial l} = 0 \Leftrightarrow \gamma(l_0 - l)^{\eta - 1} = \mu w$$

Combining the two first-order conditions yields

$$\begin{aligned}\gamma(l_0 - l)^{\eta-1} &= w \Leftrightarrow \\ \gamma(l_0 - l) &= w^{\frac{1}{\eta-1}} \Leftrightarrow \\ h &= \left(\frac{w}{\gamma}\right)^{\frac{1}{\eta-1}}\end{aligned}$$

We see that the hours choice is increasing in the hourly wage rate and decreasing in the disutility parameter γ since $\eta > 1$. We notice that the second-order condition is negative, i.e. $-\gamma(\eta - 1)(l_0 - l)^{\eta-2} < 0$ for $\eta > 1$, so we have found a maximum.

Consumption is given by

$$c = wh = w\left(\frac{w}{\gamma}\right)^{\frac{1}{\eta-1}} = w^{\frac{\eta}{\eta-1}}\gamma^{\eta-1}$$

2. The Marshallian elasticity is given by

$$\begin{aligned}
\frac{\partial h}{\partial w} &= \frac{1}{\eta-1}\frac{1}{\gamma}\left(\frac{w}{\gamma}\right)^{\frac{1}{\eta-1}-1} \Leftrightarrow \\
\frac{\partial h}{\partial w}\frac{w}{h} &= \frac{1}{\eta-1}\frac{1}{\gamma}\left(\frac{w}{\gamma}\right)^{\frac{1}{\eta-1}-1}\frac{w}{h} \Leftrightarrow \\
\frac{\partial h}{\partial w}\frac{w}{h} &= \frac{1}{\eta-1}\left(\frac{w}{\gamma}\right)^{\frac{1}{\eta-1}}\frac{1}{h} \Leftrightarrow \\
\frac{\partial h}{\partial w}\frac{w}{h} &= \frac{1}{\eta-1}
\end{aligned}$$

This means that when the wage is increases by 1 percent, then the labor supply is increased by $\frac{1}{\eta-1}$ percent.

3. The hours choice under taxation with the tax rate τ is given by $h = \left(\frac{(1-\tau)w}{\gamma}\right)^{\frac{1}{\eta-1}}$. Let the elasticity be denoted $\varepsilon = \frac{1}{\eta-1}$. We see that the reduction in hours is larger, the larger ε is. In this case, where the utility function is quasi-linear, the Marshallian elasticity is equal to the Hick's elasticity since income effects are zero. This implies that the elasticity is the same whether the individual faces taxes or not.

4. a) The optimal choice is $h_1 = \left(\frac{w}{\gamma}\right)^{\frac{1}{\eta-1}}$. b) The optimal choice is $h_2 = \left(\frac{(1-\tau)w}{\gamma}\right)^{\frac{1}{\eta-1}}$.

   c) First, h_1 cannot be optimal: The tax rate would be binding and reduce labor supply below h_1. Second, h_2 cannot be optimal. At this rate worker would not be taxed and it would be optimal to increase labor supply. Consequently labor supply must be in between the two. This means that

$$h_3 = \frac{z^*}{w}$$

   or in words that the worker chooses to locate in the kink of the piece-wise linear tax system.

5.

$$\begin{aligned}
z^* &= wh_1 \Leftrightarrow \\
z^* &= w\left(\frac{w}{\gamma}\right)^{\frac{1}{\eta-1}} \Leftrightarrow \\
\left(\frac{z^*}{w}\right)^{\eta-1} &= \frac{w}{\gamma} \Leftrightarrow \\
\gamma &= \frac{w}{\left(\frac{z^*}{w}\right)^{\eta-1}} \Leftrightarrow \\
\gamma &= \frac{w^{\eta}}{(z^*)^{\eta-1}}
\end{aligned}$$

$$
\begin{aligned}
z^* &= wh_2 \Leftrightarrow \\
z^* &= w\left(\frac{(1-\tau)\,w}{\gamma}\right)^{\frac{1}{\eta-1}} \Leftrightarrow \\
\left(\frac{z^*}{w}\right)^{\eta-1} &= \frac{(1-\tau)\,w}{\gamma} \Leftrightarrow \\
\gamma &= \frac{(1-\tau)\,w}{\left(\frac{z^*}{w}\right)^{\eta-1}} \Leftrightarrow \\
\gamma &= \frac{(1-\tau)\,w^{\eta}}{(z^*)^{\eta-1}}
\end{aligned}
$$

Individuals with γ in between $\frac{(1-\tau)w^{\eta}}{(z^*)^{\eta-1}}$ and $\frac{w^{\eta}}{(z^*)^{\eta-1}}$ will bunch at the kink point, z^*. We see that the lower bound is decreasing in the tax rate, which implies that the width of the interval increases in τ. Hence, ceteris paribus the larger kink, the more individuals will bunch. The intuition is that with a higher τ, the difference between the taxed and non-taxed incomes is larger, and more individuals with lower disutility parameter γ will also decide to bunch at the kink point, z^*.

2 The matching model with search intensity

1. The probability of getting a job is given by

$$
\begin{aligned}
\lambda(s_i) &= s_i \frac{M(sU,V)}{sU} \\
&= s_i \frac{M(sU,V)}{V}\frac{V}{sU} \\
&= \frac{s_i}{s}\theta m\left(\frac{\theta}{s}\right)
\end{aligned}
$$

where $m\left(\frac{\theta}{s}\right) \equiv M\left(\frac{s}{\theta},1\right) = \frac{M(sU,V)}{V}$ and where we have used that the matching function exhibits constant returns to scale. The job arrival rate is increasing in the labor market tightness and the worker's own search intensity, but is declining in the other workers search intensity. The latter effect is a congestion effect.

2. Inserting the job arrival rate from the previous question in the Bellman equation for an unemployed, we obtain

$$
\begin{aligned}
rV_u(s_i) &= b - c(s_i) + \lambda(s_i)\left[V_e - V_u(s_i)\right] \\
&= b - c(s_i) + \frac{s_i}{s}\theta m\left(\frac{\theta}{s}\right)\left[V_e - V_u(s_i)\right]
\end{aligned}
$$

Differentiating this with respect to s_i and setting $\frac{\partial V_u(s_i)}{\partial s_i} = 0$ yields

$$c'(s_i) = \frac{1}{s}\theta m\left(\frac{\theta}{s}\right)[V_e - V_u(s_i)]$$

where $V_u(s_i)$ is evaluated at the optimal s_i. The l.h.s. is the marginal cost of search, whereas the r.h.s. is the marginal benefit of search. The first part of the marginal benefit is the probability of getting a job offer for a marginal increase in s_i, whereas the second part is the gain in the discounted utility of getting a job. It is optimal for the worker to equate the marginal cost of search and the marginal benefits of search.

3. We are only interested in a symmetric equilibrium, where all unemployed workers are searching with the same intensity $s_i = s$. Hence, we will evaluate the first-order condition for s_i in $s_i = s$.

$$\begin{aligned} c'(s) &= \frac{1}{s}\theta m\left(\frac{\theta}{s}\right)[V_e - V_u(s)] \Leftrightarrow \\ sc'(s) &= \theta m\left(\frac{\theta}{s}\right)\frac{\gamma}{1-\gamma}\Pi_e \Leftrightarrow \\ sc'(s) &= \theta m\left(\frac{\theta}{s}\right)\frac{\gamma}{1-\gamma}\frac{h}{m\left(\frac{\theta}{s}\right)} \Leftrightarrow \\ sc'(s) &= \frac{\gamma}{1-\gamma}\theta h \qquad (1) \end{aligned}$$

We see that as the the number of vacancies (or the labor market tightness) increases, so does the search effort since $c'(s)$ is assumed to be increasing s. The positive relationship between s and θ reflects that there are positive externalities between groups (the so-called thick-market externality), i.e. an additional vacancy increases the rate at which workers find job for a given search intensity s. This means that a higher labor market tightness increases the returns to search and, therefore, also the search intensity. A higher labor market tightness also imply a higher outside option for the worker, which will increase wage and, thereby the returns to search and through this also the search intensity.

4. Free-entry in vacancy creation implies that the expected profits of having a vacancy is competed down to zero. Using the free-entry condition and the Bellman equation for a vacancy, we can write

$$\begin{aligned} r\Pi_v &= -h + m\left(\frac{\theta}{s}\right)(\Pi_e - \Pi_v) \Leftrightarrow \\ \Pi_e &= \frac{h}{m\left(\frac{\theta}{s}\right)} \end{aligned}$$

Using the Bellman equation for a filled job, we get

$$\begin{aligned} r\Pi_e &= y - w + q\left(\Pi_v - \Pi_e\right) \\ \Pi_e &= \frac{y-w}{r+q} \end{aligned}$$

Combining these two equation to eliminate Π_e, we obtain the vacancy supply curve

$$\frac{h}{m\left(\frac{\theta}{s}\right)} = \frac{y-w}{r+q} \tag{2}$$

The l.h.s. is the expected cost having a vacancy since h is the flow cost and $\frac{1}{m\left(\frac{\theta}{s}\right)}$ is the expected duration of a vacancy, whereas the r.h.s. is the expected profits of having a filled job.

5. Inserting the wage equation in the vacancy supply curve in equation (2) gives us

$$\frac{h}{m\left(\frac{\theta}{s}\right)} = \frac{y-w}{r+q} \Leftrightarrow$$

$$\frac{h}{m\left(\frac{\theta}{s}\right)} = \frac{y - \left\{\left[z - c\left(s\right)\right] + \left(y - \left[z - c\left(s\right)\right]\right)\frac{\gamma\left[r+q+\theta m\left(\frac{\theta}{s}\right)\right]}{r+q+\gamma\theta m\left(\frac{\theta}{s}\right)}\right\}}{r+q} \Leftrightarrow$$

$$\frac{h}{m\left(\frac{\theta}{s}\right)} = \frac{\left(y - \left[z - c\left(s\right)\right]\right)\frac{r+q+\gamma\theta m\left(\frac{\theta}{s}\right) - \gamma\left[r+q+\theta m\left(\frac{\theta}{s}\right)\right]}{r+q+\gamma\theta m\left(\frac{\theta}{s}\right)}}{r+q} \Leftrightarrow$$

$$\frac{h}{m\left(\frac{\theta}{s}\right)} = \frac{\left(y - \left[z - c\left(s\right)\right]\right)\frac{(1-\gamma)(r+q)}{r+q+\gamma\theta m\left(\frac{\theta}{s}\right)}}{r+q} \Leftrightarrow$$

$$\frac{h}{m\left(\frac{\theta}{s}\right)} = \frac{\left(1-\gamma\right)\left(y - \left[z - c\left(s\right)\right]\right)}{r+q+\gamma\theta m\left(\frac{\theta}{s}\right)} \Leftrightarrow$$

$$\left(1-\gamma\right)\left(y - \left[z - c\left(s\right)\right]\right) = \gamma h\theta + \frac{r+q}{m\left(\frac{\theta}{s}\right)}h \Leftrightarrow$$

$$c\left(s\right) = \frac{\gamma h}{1-\gamma}\left[\theta + \frac{r+q}{\gamma m\left(\frac{\theta}{s}\right)}\right] - \left(y - z\right) \tag{3}$$

First, we total differentiate equation (1) with respect to s and θ

$$\begin{aligned} \left[c'\left(s\right) + sc''\left(s\right)\right]ds &= \frac{\gamma h}{1-\gamma}d\theta \Leftrightarrow \\ \frac{ds}{d\theta} &= \frac{\gamma h}{1-\gamma}\frac{1}{c'\left(s\right) + sc''\left(s\right)} > 0 \end{aligned}$$

Next, we total differentiate equation (3) with respect to s and θ

$$c'(s)\,ds = \frac{\gamma h}{1-\gamma}\left[1 - \frac{(r+q)\,m'\left(\frac{\theta}{s}\right)\frac{1}{s}}{\gamma\left[m\left(\frac{\theta}{s}\right)\right]^2}\right]d\theta + \frac{\gamma h}{1-\gamma}\frac{(r+q)\,\frac{\theta}{s^2}m'\left(\frac{\theta}{s}\right)}{\gamma\left[m(\theta)\right]^2}ds \Leftrightarrow$$

$$\left[c'(s) - \frac{\gamma\theta h}{(1-\gamma)\,s}\frac{(r+q)\,m'\left(\frac{\theta}{s}\right)\frac{1}{s}}{\gamma\left[m(\theta)\right]^2}\right]ds = \frac{\gamma h}{1-\gamma}\left[1 - \frac{(r+q)\,m'\left(\frac{\theta}{s}\right)\frac{1}{s}}{\gamma\left[m\left(\frac{\theta}{s}\right)\right]^2}\right]d\theta \Leftrightarrow$$

$$\frac{ds}{d\theta} = \frac{\gamma h}{1-\gamma}\left[1 - \frac{(r+q)\,m'\left(\frac{\theta}{s}\right)\frac{1}{s}}{\gamma\left[m\left(\frac{\theta}{s}\right)\right]^2}\right]\frac{1}{c'(s) - \frac{\gamma\theta h}{(1-\gamma)s}\frac{(r+q)m'\left(\frac{\theta}{s}\right)\frac{1}{s}}{\gamma[m(\theta)]^2}}$$

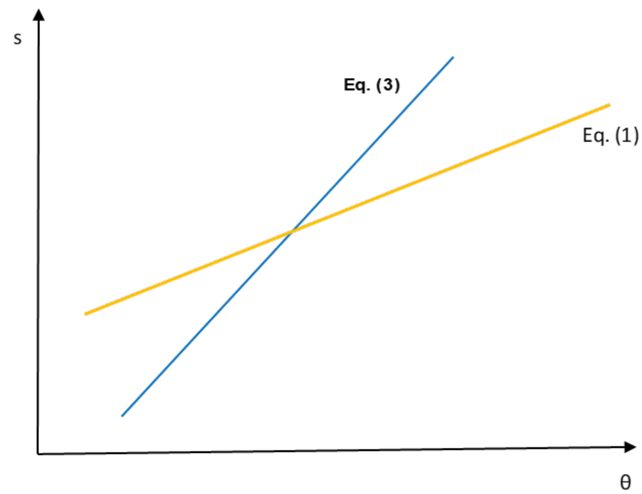
Next, we use that locally around the intersection point, it approximately holds that

$$\begin{aligned} sc'(s) &= \frac{\gamma}{1-\gamma}\theta h \Leftrightarrow \\ c'(s) &= \frac{\gamma\theta h}{(1-\gamma)\,s} \end{aligned}$$

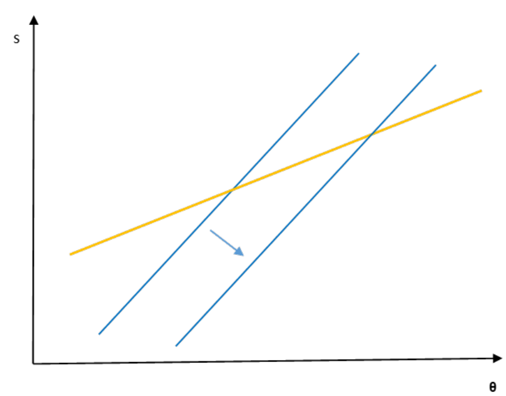
Inserting this gives us

$$\begin{aligned} \frac{ds}{d\theta} &= \frac{\gamma h}{1-\gamma}\left[1 - \frac{(r+q)\,m'\left(\frac{\theta}{s}\right)\frac{1}{s}}{\gamma\left[m\left(\frac{\theta}{s}\right)\right]^2}\right]\frac{1}{c'(s)\left[1 - \frac{(r+q)m'\left(\frac{\theta}{s}\right)\frac{1}{s}}{\gamma[m(\theta)]^2}\right]} \\ &= \frac{\gamma h}{1-\gamma}\frac{1}{c'(s)} > 0 \end{aligned}$$

First, it is clear that both equations give rise to an increasing relationship between the search intensity and the labor market tightness (for equation (3), at least this holds locally around the point where equation (1) and equation (3) intersect). Second, we see that equation (1) is less steep in (θ, s) space than equation (3) because $sc''(s) > 0$. Since the slope of the latter curve is everywhere steeper than the former, we have a unique crossing between the two curves and, hence, a unique solution for s and θ.



6. We begin by considering the figure from the previous question. This figure determines the equilibrium values of the labor market tightness and the search intensity of the workers. We see that a productivity increase shifts the curve for equation (3) downwards in (θ, s) space, whereas the first-order condition for the search intensity, i.e. equation (1), is not shifted. This means that in the new equilibrium, both the labor market tightness and the search intensity have increased. The intuition is that with a higher productivity, the match surplus increases. Hence, firms are more willing to create vacancies and workers search harder.



Next, we turn to the wage equation. The direct effect of a higher productivity is higher wages since the match surplus is higher. Furthermore, the higher labor market tightness increases the wage by strengthening the workers' bargaining position, but at the same time the higher search effort lowers the value of being unemployed, which tends to lower the wage. Hence, in principle the effect of productivity is ambiguous, but the overall effect on wages is most likely to be strongly positive.

Finally, the equilibrium unemployment rate is determined, where the Beveridge curve intersects the labor market tightness in (u, v) space. The Beveridge curve shifts inward due to the higher search effort, whereas the labor market tightness increases. This implies that the unemployment decreases whereas the effect on the vacancy rate is ambiguous.